

Expanding F-theory

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ABSTRACT: We construct a general class of new time dependent solutions of non-linear σ -models coupled to gravity. These solutions describe configurations of expanding or contracting codimension two solitons which are not subject to a constraint on the total tension. The two dimensional metric on the space transverse to the defects is determined by the Liouville equation. This space can be compact or non-compact, and of any topology. We show that this construction can be applied naturally in type IIB string theory to find backgrounds describing a number of 7-branes larger than 24.

KEYWORDS: F-Theory, p-branes, Sigma Models.

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1. Introduction

It is well known that point particles in pure 2+1 dimensional gravity — or relativistic codimension two objects in higher dimensional theories — generate a conical space with deficit angle $\alpha = m$, where m is the mass of the particle (we set $8\pi G_N = 1$) [1]. If the metric is static, the two dimensional space transverse to the defects has zero curvature away from the singularities and is non-compact when the total mass of the defects is less than 2π , or compact and of spherical topology when the total mass is exactly 4π .

In [2] it was shown that these constraints can be relaxed by allowing the metric to be time dependent. The space remains flat outside the defects, but is naturally foliated by two dimensional locally hyperbolic slices. The constant negative curvature of this two dimensional space accommodates defects with any total mass, and allows it to have any topology (subject to some mild restrictions). Physically these solutions describe 2+1 dimensional cosmologies uniformly expanding from (or contracting towards) a big bang at $t = 0$.

In this note we will make use of this fact to find expanding solutions for codimension two σ -model solitons with tension and topology not allowed by a static ansatz. This is possible because the σ -model solitons (unlike abelian Higgs model vortices, for example) have a scale invariance which, like distributions of pressureless point masses, allows them to expand uniformly (see also [3]).

Our interest in these types of configurations arises from applications in string theory where σ -models are ubiquitous. In particular we will show that this construction allows us to generalize the stringy cosmic strings of [4] to arbitrary numbers of defects and general

transverse topology. This in turn can be exploited to find F-theory [5] backgrounds of type IIB string theory with a number of 7-branes larger than 24.

Our solutions are reminiscent of those of refs. [6–9] (see [10] for a review and additional references), which considered time dependent solutions obtained by orbifolding flat space foliated with negatively curved slices. In our case the presence of sources means that the transverse metric does not in general have constant negative curvature, but in certain limits the sources are point-like and our solutions are locally flat. It would be interesting to investigate the connection with these works.

2. Construction

Our results follow in part from [2], which constructed new gravitational solutions corresponding to an arbitrary number of point-like codimension two objects coupled to gravity. To see how this works, recall that a codimension two defect which is boost invariant along its world volume directions — such as a straight relativistic cosmic string — generates a locally flat metric with a conical singularity at the location of the defect. Choosing a metric ansatz

$$ds^2 = -dt^2 + dx_i dx^i + e^{\phi(z, \bar{z})} dz d\bar{z} \tag{2.1}$$

appropriate for a distribution of parallel defects, the vacuum Einstein’s equations reduce to the Poisson’s equation for ϕ , with δ -function sources:

$$\partial_z \partial_{\bar{z}} \phi = \sum_{i=1}^N m_i \delta^2(z - z_i). \tag{2.2}$$

Since the curvature R_2 of the two dimensional space Σ parametrized by (z, \bar{z}) is related to ϕ by $\sqrt{\gamma} R_2(\gamma) = -2i \partial_z \partial_{\bar{z}} \phi$, eq. (2.2) implies $R_2 = 0$ away from the defects. We can apply the Gauss-Bonnet theorem to Σ :

$$\int_{\Sigma} R_2 + 2 \int_{\partial \Sigma} K_1 = 4\pi \chi, \tag{2.3}$$

where K_1 is the extrinsic curvature of the boundary and χ is the Euler character. Using eqs. (2.2) and (2.3) one finds

$$2 \int_{\partial \Sigma} K_1 = 4\pi \chi - 2 \sum_{i=1}^N m_i. \tag{2.4}$$

Since a surface of genus g has $\chi = 2 - 2g$, if the space is compact and without boundary the only solution has spherical topology and $\sum m_i = 4\pi$. Non-compact asymptotically conical solutions exist when $\sum m_i \leq 2\pi$.

In [2] it was shown that these restrictions on the deficit angles can be avoided if the two dimensional metric is uniformly expanding:

$$ds^2 = -dt^2 + dx_i dx^i + t^2 e^{\phi(z, \bar{z})} dz d\bar{z}. \tag{2.5}$$

The vacuum Einstein's equations become a Liouville equation for the conformal factor:

$$\partial_z \partial_{\bar{z}} \phi = \frac{1}{2} e^\phi - \sum_{i=1}^N m_i \delta^2(z - z_i). \tag{2.6}$$

This differs from the Poisson equation (2.2) by the term $\frac{1}{2}e^\phi$, which arises due to the t^2 dependence. In the absence of sources the solution of this equation, $\phi = 2 \ln \left(\frac{2i}{z-\bar{z}} \right)$, is a metric for the hyperbolic plane H_2 with constant negative curvature. The full metric is flat space written in a two dimensional hyperbolic slicing. In the presence of defects the space away from the singularities is still locally flat, but the global solution is non-trivial.

The constraint on the total mass is modified because the curvature term on the left-hand side of (2.3) now makes a negative definite contribution. Integrating the Liouville equation (2.6) and using (2.3) one finds that the volume of Σ is given by

$$V = \sum_{i=1}^N m_i + 4\pi(g - 1) \tag{2.7}$$

(from here on we will drop the boundary term for simplicity). Positivity of the volume implies that there are compact solutions with spherical topology only if $\sum m_i > 4\pi$, and (at least for closed and compact spaces) in general $V > 0$ is the only constraint on the existence of solutions for all topologies (see theorem A of [12]).

2.1 Sigma model solitons

Consider the following action

$$S = - \int \sqrt{-g} d^d x \left(\frac{1}{2} R + K_{\tau_i \bar{\tau}_j} \partial_\mu \tau_i \partial^\mu \bar{\tau}_j \right) \tag{2.8}$$

describing a complex non-linear σ -model coupled to gravity. Here the fields τ_i are complex scalars which could arise as moduli of a compactification from higher dimension, and $K_{\tau_i \bar{\tau}_j} \equiv \partial_{\tau_i} \partial_{\bar{\tau}_j} K$ determines the metric of the target space complex manifold. In $d = 4$ this is the bosonic part of a supersymmetric action.

Under some topological conditions actions of the form (2.8) admit soliton solutions describing codimension two objects [11]. To see this one can choose the metric ansatz (2.5), and assume the scalars depend only on the transverse coordinates: $\tau_i = \tau_i(z, \bar{z})$. The two dimensional conformal factor $t^2 \phi(z, \bar{z})$ is time dependent, but nonetheless it cancels in the equations of motion for the scalars:

$$K_{\tau_i \bar{\tau}_k} (\partial_z \tau_i \partial_{\bar{z}} \tau_l + \partial_{\bar{z}} \tau_i \partial_z \tau_l) + 2K_{\tau_i \bar{\tau}_k} \partial_z \partial_{\bar{z}} \tau_i = 0. \tag{2.9}$$

These equations are trivially solved by any holomorphic (or anti-holomorphic) functions $\tau_i(z)$, but one should make sure that the solution is well defined on the entire manifold spanned by the scalars. The solutions so obtained correspond to a holomorphic mapping of the spacetime surface Σ into the target space manifold. The energy can be expressed in a Bogomol'nyi-like form as the integral of the Kähler (1,1) form:

$$E = -\frac{i}{2} \int d^2 z \partial_z \partial_{\bar{z}} K(\tau_i(z), \bar{\tau}_j(\bar{z})) = -\frac{1}{2} \int d^2 \tau_1 \dots d^2 \tau_n \sqrt{\det(K_{\tau_i \bar{\tau}_j})}. \tag{2.10}$$

The right-hand side is essentially the volume of the target space times an integer N counting the degree of the mappings $\tau_i(z)$. The energy E is positive if the Kähler metric is positive definite (as required by supersymmetry), and with our normalization corresponds to the total deficit angle.

Having solved the scalar equations of motion, we need to show that Einstein's equations can be solved consistently. With the time dependent ansatz (2.5) they reduce to a single equation:

$$\partial_z \partial_{\bar{z}} \phi = \frac{1}{2} e^\phi - \partial_z \partial_{\bar{z}} K(\tau_i(z), \bar{\tau}_j(\bar{z})), \tag{2.11}$$

which is similar to (2.6) with the δ -function sources replaced by the smooth energy density $\partial_z \partial_{\bar{z}} K$.

As in the case of point particles, some mild restrictions on the solutions of eq. (2.11) arise depending on the total energy of the source. Integrating the Liouville equation and using the definition of E we obtain

$$V = E + 4\pi(g - 1) \tag{2.12}$$

(cf. eq. (2.7)). In the static ansatz (2.1) the left-hand side of eq. (2.12) would be zero, and the only allowed compact manifold would be spherical with energy $E = 4\pi$. The expanding ansatz allows much more general solutions. For spherical topology the total energy should again be greater than 4π , and for higher genus surfaces there is evidently no constraint from Gauss-Bonnet (at least if $E > 0$).

Physically we expect solutions of (2.11) to exist so long as the right-hand side of (2.12) is positive. As we mentioned previously, existence has been proven when the sources are δ -functions [12]. In the smooth case the results of [13] prove existence for arbitrary topology of the transverse space Σ , at least when it is closed, compact, $\chi < 0$, and the energy density $\partial_z \partial_{\bar{z}} K > 0$. This last is guaranteed if the Kähler metric on the target space is positive. The situation is more complex when $\chi \geq 0$.

2.2 CP_1

As an explicit example, we can apply the construction outlined above to a σ -model with CP_1 target space [14]. The Kähler potential is

$$K = a^2 \log[1 + \tau \bar{\tau}], \tag{2.13}$$

so that the metric is that of a round sphere. The topological solitons of this model correspond to mappings of the spacetime sphere into the target space sphere. The simplest example of such a map is simply $\tau = z$. From direct integration of (2.10) we see that the energy of this configuration is $2\pi a^2$. The general N -vortex solution is given in terms of a rational function,

$$\tau(z) = \frac{P(z)}{Q(z)} \tag{2.14}$$

where $P(z)$ and $Q(z)$ are polynomials of degree p and q without common factors. The energy (2.10) of these solutions can be easily evaluated by noting that the mapping (2.14)

covers the target space sphere $N=\max(p, q)$ times, so that

$$E = 2\pi a^2 N. \tag{2.15}$$

An explicit solution of the Liouville equation can be found when $N = 1$. Since this is a trivial mapping from sphere to sphere, the natural guess is

$$e^\phi = \kappa \frac{1}{(1 + z\bar{z})^2}. \tag{2.16}$$

By plugging into eq. (2.11) one finds that this is a solution if

$$\kappa = 2a^2 - 4. \tag{2.17}$$

From the fact that the metric must be positive it follows that $a^2 > 2$, which is nothing but the constraint implied by the positivity of the volume in eq. (2.12).

3. F-theory revisited

Arguably the most interesting application of our time dependent solutions is in the context of string theory. The simplest case is type IIB string theory, where the relevant σ -model is provided by the axion-dilaton, which spans an $SL(2, \mathbb{R})/U(1)$ manifold. Static codimension two configurations with varying axion-dilaton are the starting point for F-theory [5].

At the supergravity level the relevant part of the action is

$$S = - \int \sqrt{-g} d^{10}x \left(\frac{1}{2}R - \frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{(\tau - \bar{\tau})^2} \right), \tag{3.1}$$

which has the form of (2.8) with $K = -\log[-i(\tau - \bar{\tau})]$. Topological solitons of this theory corresponding to mappings of the Riemann sphere into the target space manifold were considered in [4]. To construct solutions with finite energy one needs to exploit the symmetries of the theory by allowing τ to undergo non trivial $SL(2, \mathbb{Z})$ monodromies as it varies on the compactification manifold.¹ Since τ can be interpreted as the modular parameter of a “hidden torus” associated with a 12 dimensional interpretation of the theory, the solutions so constructed describe an elliptically fibered manifold obtained by erecting a torus at each point on the base manifold [5].

To construct these configurations more explicitly it is convenient to parametrize the torus as the complex surface

$$y^2 = x^3 + fx + g \tag{3.2}$$

embedded in \mathbb{C}^2 . The complex numbers f and g determine the complex structure of the torus:

$$j(\tau) = \frac{4(24f)^3}{27g^2 + 4f^3}, \tag{3.3}$$

¹At the classical level the action (3.1) is $SL(2, \mathbb{R})$ invariant, but quantum effects break this to the discrete subgroup $SL(2, \mathbb{Z})$. This is manifest in the twelve dimensional picture, where the symmetry is the modular transformation of the complex structure τ on the torus.

where $j(\tau)$ is the modular function mapping the fundamental domain of $SL(2, \mathbb{Z})$ to the Riemann sphere. Multi-vortex solutions are obtained by taking f and g to be polynomials in the coordinate z of the base manifold. The zeros of the polynomial

$$\Delta = 27g(z)^2 + 4f(z)^3 \tag{3.4}$$

determine points where the torus degenerates, but where the space is nonetheless regular [4]. Noting that the area of the fundamental domain of $SL(2, \mathbb{Z})$ is $\pi/6$ in our conventions, it follows immediately from eq. (2.10) that the total energy is

$$E = N \frac{\pi}{6}, \tag{3.5}$$

where N is the degree of Δ .

In the type IIB context these configurations carry (p, q) 7-brane charge [5, 16]. For the case relevant to the simplest F-theory compactifications the base of the elliptic fibration is the Riemann sphere, therefore from eqs. (3.5) and (2.12) it follows that it takes precisely 24 7-branes to close the space. The most general solution of this type is obtained by taking $f(z)$ and $g(z)$ to be polynomials of degree 8 and 12 respectively. The manifold so constructed is an elliptic fibration of K_3 , and the metric on the base manifold can be found explicitly by solving the Poisson equation sourced by the σ -model [4].

When the number of 7-branes is greater than 24 they cannot be placed on a sphere without some source of negative curvature. Using the time dependent ansatz (2.5), however, the problem is of the form we discussed in the previous section. The solution for the scalars is the same as eq. (3.3), while the metric is now determined by the associated Liouville equation (2.11).

The Gauss-Bonnet theorem would require at least $N=25$ defects to find a compact solution. However in this case there will be branch cut singularities in $\tau(z)$ [4, 15]. For this reason we will focus on the case $N = 6n$, as in these cases the solution is regular. As we show below, in a certain limit this reduces to the Liouville equation with point-like singularities.

The number of parameters necessary to specify our expanding F-theory solutions is similar to the dimension of the moduli space of static 7-branes. For the case where the number of singularities is $6n$ the polynomials f and g have degree $2n$ and $3n$. This corresponds to $5n + 2$ complex free parameters. Taking into account conformal transformations and the fact that the solution only depends on the ratio f^3/g^2 we conclude that the general solution with $6n$ singularities depends on $5n - 2$ complex parameters. Note however that, contrary to the F-theory case, the volume of the internal space (at given t) is now fixed by eq. (2.12).

3.1 Sen's limit

A general concern about solutions with varying axion-dilaton is that the classical configuration might be strongly corrected. This seems a particularly acute worry for our time dependent configurations because (as we will discuss) supersymmetry is broken. Since the

imaginary part of τ controls the string coupling, it follows from eq. (3.3) that there are regions of space where the string coupling is large and perturbation theory cannot be applied even in principle. While the topological origin of the solutions strongly hints that they will survive in the full theory, it is useful to find a limit where these solutions are fully under control.

In [17] Sen considered a special limit of F-theory where τ becomes constant and the solution can be understood as an orientifold of type IIB string theory. We now show that the same limit can be used in the time dependent case. To obtain a constant axion-dilaton one must go to a particular limit of the parameter space of our solutions where f, g in (3.3) are chosen so that $f^3/g^2 = \alpha$. The free parameter α controls the string coupling, and can be chosen so that the coupling is small. The energy E of the solution in the limit is still determined by the degree of the polynomial $27g^2 + 4f^3$.

In this limit the σ -model solitons collapse to point-like singularities corresponding to a deficit angle of π each [17]. For the static F-theory solutions there are four stacks of six (p, q) 7-branes each. The manifold is topologically a sphere whose metric,

$$ds_2^2 = R^2 \prod_{m=1}^4 |z - z_i|^{-1/2} \tag{3.6}$$

is locally flat but with four singularities. Similarly, if the total number of singularities is $N = 6n$ with $n \geq 5$ we can pack them into stacks of six each. The metric is then determined by the Liouville equation with δ -sources (2.6). Around the singularities the metric is just a reparametrization of flat space with conical deficit. As a consequence only globally it is possible to distinguish these solutions from the ones in the usual F-theory. As in [17] by looking at the transformation of the coordinates (x, y) of the torus (3.2) moving around the singularities one discovers that there exists a non-trivial $SL(2, \mathbb{Z})$ transformation

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \tag{3.7}$$

under which τ is invariant.

In the perturbative string theory framework such a monodromy is associated with the presence of an orientifold plane so that each singularity is described perturbatively by an $O7$ -plane and four 4 7-branes [17]. Since our construction allows an arbitrary number of defects and locally the space is the same as in [17], it is natural to interpret our solutions as an orientifold of string theory with five or more $O7$ -planes. In particular this might allow one to give a perturbative world-sheet description of these solutions.²

Another interesting feature of this solutions is that on a surface with constant negative curvature 2π conical defects (known as parabolic singularities) become possible. Close to such a singularity the metric is given by

$$e^\phi \sim \frac{1}{z\bar{z}(\log z\bar{z})^2}. \tag{3.8}$$

²We should note that since some of the fields transform under the full $SL(2, \mathbb{Z})$ or its double cover, global obstructions may in fact require $N = 24n$. We thank Simeon Hellerman for pointing this out to us.

The proper distance from the singularity at $z = 0$ to any finite z diverges, but the volume is finite. Evidently moving two π singularities together draws that point out into a cusp. The $SL(2, \mathbb{Z})$ transformation around these points is trivial. It would be interesting to study the behavior of string theory close to these singularities and see whether tachyon instabilities appear along the lines in ref. [19].³

3.2 Other topologies

With the static ansatz the only allowed topology is spherical, but as we have seen these constraints are avoided when the ansatz is time dependent. From eq. (2.12) we see that there is no topological constraint for $g > 1$, while the toroidal topology simply needs $E > 0$.

We can find generalizations of the solutions of [4] to cases where the base of the fibration is a Riemann surface of genus g by considering mappings between the fundamental region of $SL(2, \mathbb{Z})$ and the Riemann surface. We can construct such maps by composing the inverse of the j -function with a map from the Riemann surface into the Riemann sphere. In particular, if $z = h(\zeta)$ is a meromorphic map from a Riemann surface Σ with coordinate ζ into the complex plane, then $\tau(\zeta) = j^{-1}(h(\zeta))$ describes a “stringy cosmic string” configuration with transverse space Σ .

This can be done rather explicitly for the case of toroidal topology. In this case the functions f and g in (3.3) must be meromorphic functions defined on the base torus, i.e. elliptic functions. These are in general rational functions of the Weierstrass \wp -function (with periods determined by the torus) and its derivative. The simplest solution is given by

$$j(\tau(z)) = \frac{4(24\wp(z))^3}{27 + 4\wp(z)^3}. \tag{3.9}$$

Since \wp has one double pole at $z = 0$ and two single zeros in the fundamental region of the spacetime torus (not to be confused with the “hidden torus” of the 12 dimensional description) one can check that τ does not have orbifold singularities and is well defined on the torus. The mapping (3.9) covers the fundamental region of $SL(2, \mathbb{Z})$ six times (this follows from the fact that the $\wp(z)$ is a double cover of the Riemann sphere), so evidently this configuration can be understood as six 7-branes on an expanding torus. We can find an analog of the Sen limit that should connect these solutions to perturbative string theory. This could be done for example by taking $f = \alpha\wp^2$ and $g = \wp^3$. It would be interesting to study these configurations more in detail.

We can also find solutions in cases where the transverse space is non-compact. A simple example follows from the above: given a solution with toroidal topology, we can always re-interpret the configuration as an infinite expanding array of defects. More general configurations should also exist, either with a finite or infinite number of defects. In the case where the total mass is finite, the extrinsic curvature term in eq. (2.4) compensates for the bulk curvature contribution.

³We would like to thank Allan Adams for discussions on this point.

4. Corrections

F-theory backgrounds constructed from elliptic fibrations of Calabi-Yau manifolds are supersymmetric [5, 20]. In the expanding case the time dependence implies that supersymmetry is broken. It is interesting to see in detail how this happens. For simplicity we consider the problem in $4D$ with four supercharges following [18]. For a supersymmetric σ -model coupled to gravity the variations of the fermions are

$$\begin{aligned}\delta_\epsilon \chi_i &= i\sqrt{2}\sigma^\mu \bar{\epsilon} \partial_\mu \tau_i \\ \delta_\epsilon \psi_\mu &= \partial_\mu \epsilon - \epsilon \omega_\mu - \frac{1}{4} (K_{\tau_j} \partial_\mu \tau_j - K_{\bar{\tau}_j} \partial_\mu \bar{\tau}_j) \epsilon\end{aligned}\tag{4.1}$$

where ω_μ is the spin connection. For holomorphic solutions of the scalars the variation of the spinors χ_i is zero for $\hat{\epsilon} = \gamma^{\bar{z}} \epsilon$. The gravitino equation implies an integrability equation,

$$[D_\mu, D_\nu] \hat{\epsilon} = 0\tag{4.2}$$

where D_μ is defined by the variation of the gravitino above. One can check that this condition is just the Liouville equation (2.11), so it is automatically satisfied by the background. However the integrability condition is necessary but not sufficient to guarantee the existence of Killing spinors. In fact writing explicitly the variations of the gravitino, due to the time dependence of the ansatz, one finds that $e^\phi \hat{\epsilon} = 0$. This proves that the expanding solutions are not supersymmetric. However, the energy density dilutes with the expansion and therefore the supersymmetry is approximately restored at late times.

In Sen's limit, the background around the singularities is identical to the supersymmetric case, so locally one can find solutions of the Killing spinor equations. There are however no globally well defined Killing spinors (except possibly in the non-compact case $N = 6$). The breaking of supersymmetry here is reminiscent of Scherk-Schwarz compactifications where supersymmetry is broken non-locally by boundary conditions.

Since the 2π singularities have no flat space analog, we expect them to break supersymmetry locally and they are likely unstable. Similarly, configurations of N strings where $N \neq 6n$ are non-supersymmetric even in flat space, and are likely to be locally unstable (but see [15] for an interesting exception in the non-compact case). Since supersymmetry is broken at loop level there will be a force between the different stacks of 7-branes. At late times this effect could just be computed from the Casimir energy in the supergravity approximation. As we approach $t = 0$ this will presumably turn in a tachyon instability similar to the one of the 2π singularities.

Finally our solutions will have both g_s and α' corrections. As we argued above the first ones become under control everywhere in the space in the Sen's limit. For the latter it is sufficient to note that the curvatures scale as,

$$R \propto 1/t^2\tag{4.3}$$

At large t all the correction due to higher powers of the curvatures become negligible. At least in the Sen's limit, the space is locally supersymmetric, the string coupling can be taken arbitrarily small, and the non-supersymmetric states become very massive, so we expect these solutions to be under good perturbative control.

5. Discussion

In this note we have presented new σ -model solutions corresponding to supermassive codimension two solitons coupled to gravity. The time dependence of the metric allows us to construct solutions with an arbitrarily large number of defects and differing transverse topologies, evading the constraints which apply to static configurations. In particular we have shown that this construction can be applied to type IIB string theory, generalizing F-theory backgrounds to an arbitrary number of 7-branes.

We have left many interesting questions unanswered. For example in F-theory, solutions can be viewed as a compactification from 12 dimensions. In particular elliptically fibered Calabi-Yau manifolds of any dimension can be used [20]. Here we have considered the simplest case where the base space is two dimensional, and the Liouville equation plays a key role. It would be interesting to consider generalizations to higher dimensional base manifolds. Another direction is to study the field theory description of these solutions using brane probes [17, 21, 22].

On the gravitational side, one could study the stability of these solutions under perturbations, for example where the defects are given some non-zero relative velocities. Perhaps the most pressing question is what happens as we approach the singularity at $t = 0$. Here higher curvature corrections will become important and the supergravity approximation breaks down. Since supersymmetry is broken instabilities are likely to appear. One possibility is that tachyon condensation could resolve the singularity. A related conjecture is that these backgrounds are dual or connected via tachyon condensation to supercritical string theories [23]. We leave these investigations to future work.

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